647510 **MSC4MatE401A** **Seat No :________**

M.Sc.(Maths) Semester - 4 (*CBCS*) Examination
April/May-2022 (NEW COURSE)
Financial Mathematics (Elective)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 (A) Attempt any *Two*: (10)

- (a) A bank pays a yearly compounded interest rate of 6%. If 100 is initially deposited, how much interest it will earned after (i) 30 days; (ii) 60 days; (iii) 120 days ?
- (b) Explain European call option with an example.
- (c) Find values of r for which the cash flow sequence 20,10 preferable to the cash flow sequence 0,34, where r is an interest rate compounded yearly.

Q-1 (B) Attempt any *One*: (04)

- (a) Discuss types of options.
- (b) Assuming continuously compounding interest rate r, what is the present value of a cash flow sequence that returns the constant amount C at each time $t_0, t_0+t, t_0+2t, \ldots$?

Q-2 (A) Attempt any *Two*: (10)

- (a) State and prove Ito's lemma.
- (b) State and prove arbitrage theorem.
- (c) Consider a November call option to buy a share for $50 with option value $2.50. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

Q-2 (B) Attempt any *One*: (04)

- (a) Discuss random walk.
- (b) Consider a variable S that follows process $dS = \mu S dt + \sigma S dz$. What is the process followed by (i) $y = 2S$; (ii) $y = S^2$.

Q-3 (A) Attempt any *Two*: (10)

- (a) Derive Black-Scholes differential equation.
- (b) Explain delta hedging with an example.
- (c) Derive formula of put-call parity.

Q-3 (B) Attempt any *One*: (04)

- (a) Discuss assumptions to derive Black-Scholes differential equation.
- (b) State Black-Scholes formulas on an asset paying no dividend.

Q-4 (A) Attempt any *Two*: (10)

(a) Derive the Black-Scholes formula on an asset paying dividend.

(b) Discuss jump condition for discrete dividend.

(c) Suppose that 2 - year interest rates in United States and Australia are 6% and 8%per annum with continuously compounding, respectively, and the spot exchange rate between US dollar to Australian dollar is 0.8 US dollars per Australian dollar. Find the value of futures contract and justify it.

Q-4 (B) Attempt any *One*: (04)

(a) Discuss American options.

(b) Discuss properties of Black-Scholes model.

Q-5 (A) Attempt any *Two*: (10)

(a) In usual notation, prove: $P(t) = e^{-\int_0^t r(s)ds}$.

(b) Let an amount A be invested for n years at the interest rate R per annum with compounding frequency m per annum. Find the value of an investment at the end of n years.

(c) Find the value of an European put option on a non-dividend paying stock when the current stock price is \$50, the strike price is \$50, the risk free interest rate is 10% per annum with continuously compounding, the volatility is 40% per annum, and the time to maturity is 3-months.

Q-5 (B) Attempt any *One*: (04)

(a) Derive the fundamental solution of the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $(t > 0, -\infty < x < \infty)$ subject to the conditions $\frac{\partial u}{\partial x}(0,t) = 0, u(0,t) = \frac{1}{\sqrt{2\pi t}}$.

(b) Explain: 'a long position in a European put option should be hedged by a long position of delta number of underlying asset' with an example.

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